Autoresonant transition in the presence of noise and self-fields

Ido Barth[1], Lazar Friedland[1], Eli Sarid[2], and Arkadi Shagalov[3]

1. *Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem, Israel.*
2. *Department of Physics, NRCN-Nuclear Research Center Negev, Beer Sheva, Israel.*
3. *Institute of Metal Physics, Ekaterinburg, Russian Federation.*

Abstract

Autoresonance is a nonlinear phenomenon characterized by continuing phase-locking between a dynamical system and external oscillatory perturbations despite slow variation of system's parameters. A sharp threshold for autoresonant capture of an ensemble of trapped particles driven by chirped frequency oscillations is analyzed. It is shown that at small temperatures *T*, the capture probability versus driving amplitude is a smoothed step function with the step location and width scaling as $\alpha^{3/4}$ (α being the chirp rate) and $(\alpha T)^{1/2}$, respectively. Strong repulsive self-fields reduce the width of the threshold considerably, as the ensemble forms a localized autoresonant macroparticle.

Published: PRL **103,** 155001 (2009).

Introduction

A. Nonlinear oscillator

A driven, damped, noisy pendulum (dimensionless variables).

$$\ddot{x}+\nu\dot{x}+\sin x=\varepsilon\cos\psi_d+\sqrt{2\nu T}\,\xi(t)$$

- Delta correlated Gaussian noise: $\langle\xi(0)\xi(t)\rangle=\delta(t)$
- Chirped driving frequency: $\omega_d(t)=\dot{\psi}_d=1-\alpha t$ passes through linear resonance at t = 0.
- A continuing *phase-locking* is established and preserved due to nonlinearity.
- T=0 $\longrightarrow$ sharp threshold for capture into resonance for $\varepsilon>\varepsilon^{cr}$.

What is the effect of the thermal noise?

B. Trapped nonneutral plasma

Penning trap:

Driving potential

Vlasov-Poisson equations:

$$\begin{cases} f_t+uf_x-\left(V_x+\varphi_x+\varphi_x^{drive}\right)f_u=0\\ \varphi_{xx}-\chi^2\varphi=-\eta^2\int_{-\infty}^{\infty}f(u,x,t)\,du\end{cases}$$

- External potential: $V=1-\cos x$
- Driving potential: $\varphi_d=\varepsilon x\cos\psi_d$
- Self-potential: $\varphi(x)$

What is the effect of the repulsive self-field?

Theory

Assumptions

- Sufficiently small temperature $\longrightarrow$ Weak nonlinearity.
- Sufficiently fast passage through resonance $\longrightarrow$ Noise is negligible.
- However, noise and dissipation form thermally distributed, random initial conditions, f_0 (FDT).

Analysis

- Seek solution of form $x=a\cos\theta.$
- Phase mismatch: $\phi=\theta-\psi_d$
- Neglect non resonant terms **(single resonance approximation)**.
- Average over the fast phase.
- Transform to *slow* variables: $\begin{cases}\dot{a}=-\frac{\nu}{2}a-\frac{\varepsilon}{2}\sin\phi\\ \dot{\phi}=\frac{a^2}{16}-\alpha t-\frac{\varepsilon}{2a}\cos\phi\end{cases}$
- Rescale the time: $t\to\tau=\sqrt{\alpha}t.$
- Define $\gamma=\frac{\nu}{2\sqrt{\alpha}}$, $A=\frac{a}{4\alpha^{1/4}}$, and $\mu=\sqrt{\frac{3}{36}}\frac{\varepsilon}{\alpha^{3/4}}$. (Driving Parameter)
- Complex variable: $\Psi=Ae^{i\phi}$
- NLSE: $i\dot{\Psi}+\left(|\Psi|^2-\tau\right)\Psi+i\gamma\Psi=\mu$
- Phase locked asymptotic solution for $\mu>\mu_{cr}(\gamma,A_0,\phi_0)$:

$$|\Psi|^2\approx\tau,\quad \phi=const$$

- Critical driving parameter is periodic in the initial phase mismatch

$$\mu_{cr}=c_0(\gamma)+\kappa A_0\cos\phi_0$$

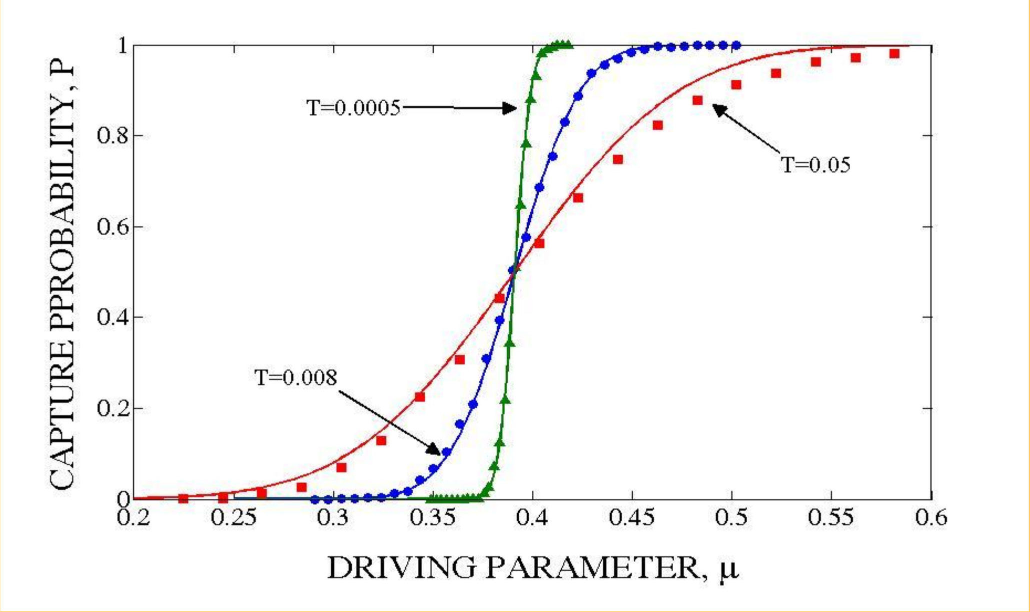
- Resonant capture probability: $P(\mu,A_0)=\begin{cases}0 & \mu<c_0-\kappa A_0\\ \frac{1}{\pi}\arccos\left(\frac{c_0-\mu}{\kappa A_0}\right) & |\mu-c_0|<\kappa A_0\\ 1 & \mu>c_0+\kappa A_0\end{cases}$
- Integration over the thermal distribution

$$P(\mu)=\int_0^{\infty}P(\mu,A_0)f_0(A_0)\,dA_0$$

Results

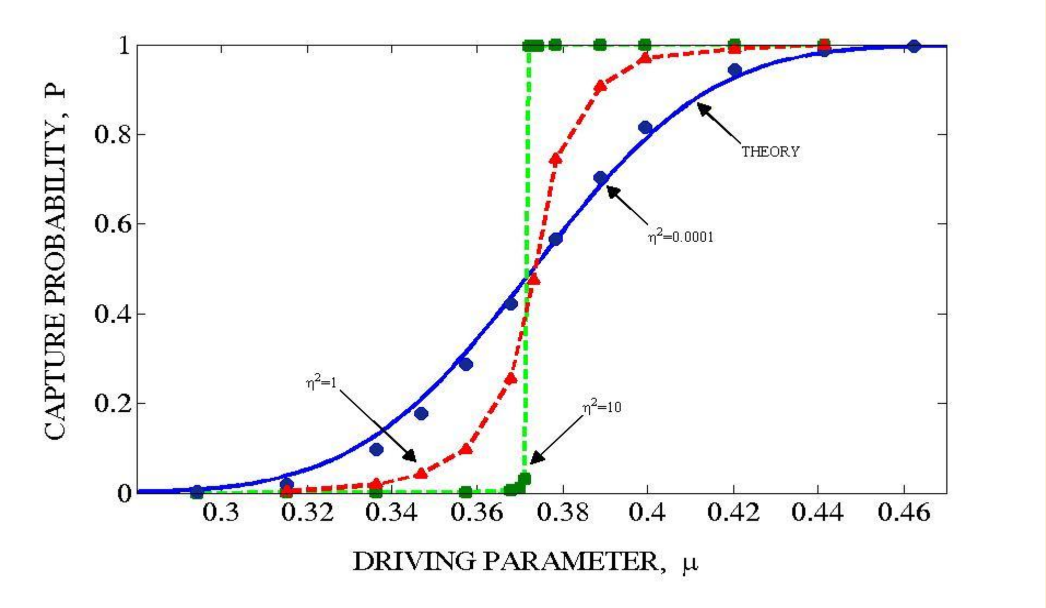
Probabilities of capture into autoresonance

A. Nonlinear oscillator



Transition width increases with temperature !

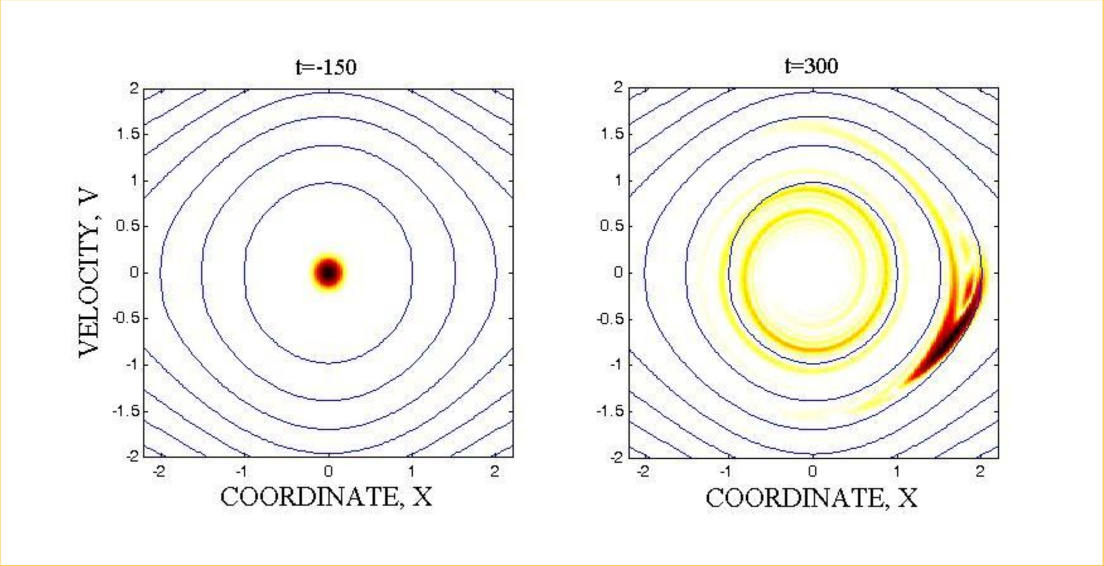
B. Trapped plasma



Repulsive self-field makes the width narrower !

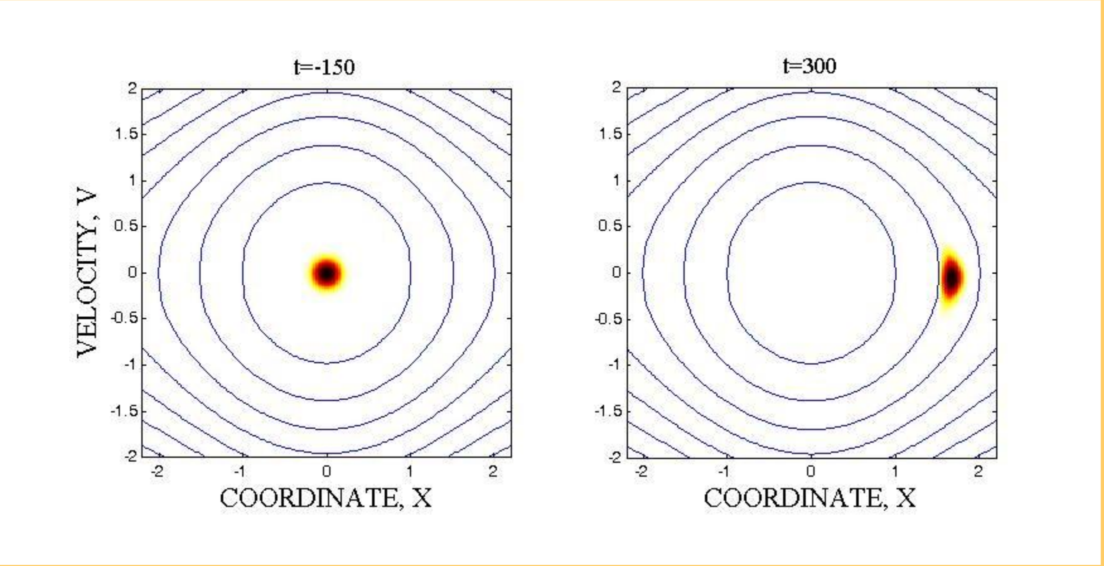
Plasma distribution in phase space

Low plasma density $(\eta^2=0.0001)$ $\mu=0.38$



57 % captured

High plasma density $(\eta^2=10)$ $\mu=0.38$



100 % captured

Autoresonant macroparticle

Conclusions

- We presented a theory describing the shape and the width of the autoresonant phase-locking transition versus the driving amplitude for both stochastic oscillators and dilute nonneutral plasmas in Penning traps driven by a chirped frequency drive.
- We observed and explained why the repulsive self-field yields a strong bunching effect and a significant narrowing of the phase-locking transition width.
- The plasma in this case behaves like a single autoresonant macroparticle and can be efficiently controlled by external parameters.
- Our theory agrees with simulations for the driven oscillator in the presence of thermal noise, as well as with Vlasov-Poisson simulations in the case of sufficiently low plasma densities.

Acknowledgment: This work was supported by the Israel Science Foundation (Grant No. 1080/06).